Warm Up Exercises

1. Find the value of $x$.

**ANSWER** 32

2. Write the converse of the following statement. If it is raining, then Josh needs an umbrella.

**ANSWER** If Josh needs an umbrella, then it is raining.
3.3 Prove Lines are Parallel

**Goal:** Use angle relationships to prove that lines are parallel.

**Key Vocabulary:**
- paragraph proof
- converse
- two-column proof
3.3 Prove Lines are Parallel

**Goal:** Use angle relationships to prove that lines are parallel.

**Postulates, Corollaries, and Theorems:**
- Postulate 16: Corresponding Angles Converse
- Theorem 3.4: Alternate Interior Angles Converse
- Theorem 3.5: Alternate Exterior Angles Converse
- Theorem 3.6: Consecutive Interior Angles Converse
- Theorem 3.7: Transitive Property of Parallel Lines
Definitions

**PARAGRAPH PROOF:** A type of proof written in paragraph form.

The statements and reasons in a paragraph proof are written in sentences, using words to explain the logical flow of the argument.
Definitions

CONVERSE: The statement formed by exchanging the hypothesis and conclusion of a conditional statement.

Statement: If $m\angle A = 90^\circ$, then $m\angle A$ is a right angle.

Converse: If $m\angle A$ is a right, then $m\angle A = 90^\circ$. 
Definitions

**TWO-COLUMN PROOF:** A type of proof written as numbered statements and corresponding reasons that show an argument in a logical order.
**POSTULATE 16: Corresponding Angles Converse**
If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.
EXAMPLE 1  Apply the Corresponding Angles Converse

ALGEBRA  Find the value of $x$ that makes $m \parallel n$.

SOLUTION

Lines $m$ and $n$ are parallel if the marked corresponding angles are congruent.

$$(3x + 5)^\circ = 65^\circ$$  Use Postulate 16 to write an equation

$3x = 60$  Subtract 5 from each side.

$x = 20$  Divide each side by 3.

The lines $m$ and $n$ are parallel when $x = 20$. 
1. Is there enough information in the diagram to conclude that \( m \parallel n \)? Explain.

**ANSWER**

Yes. \( m \parallel n \) because the angle corresponding to the angle measuring \( 75^\circ \) also measures \( 75^\circ \) since it forms a linear pair with the \( 105^\circ \) angle. So, corresponding angles are congruent and Postulate 16 says the lines are parallel.
2. *Explain why Postulate 16 is the converse of Postulate 15.*

**ANSWER**

Postulate 16 switches the hypothesis and conclusion of Postulate 15.
**THEOREM 3.4: Alternate Interior Angles Converse**

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.
Prove the Alternate Interior Angles Theorem

**GIVEN:** \( \angle 4 \cong \angle 5 \)

**PROVE:** \( g \parallel h \)

1. \( \angle 4 \cong \angle 5 \)  
   1. Given

2. \( \angle 1 \cong \angle 4 \)  
   2. Vertical Angles Congruence Theorem

3. \( \angle 1 \cong \angle 5 \)  
   3. Transitive Property of Congruence

4. \( g \parallel h \)  
   4. Corresponding Angles Converse
THEOREM 3.5: Alternate Exterior Angles Converse
If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.
Prove the Alternate Exterior Angles Theorem

**GIVEN:** \( \angle 2 \cong \angle 7 \)

**PROVE:** \( m \parallel n \)

1. \( \angle 2 \cong \angle 7 \)  
   1. Given

2. \( \angle 7 \cong \angle 6 \)  
   2. Vertical Angles Congruence Theorem

3. \( \angle 2 \cong \angle 6 \)  
   3. Transitive Property of Congruence

4. \( m \parallel n \)  
   4. Corresponding Angles Converse
THEOREM 3.6: Consecutive Interior Angles Converse
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

If \( \angle 3 \) and \( \angle 5 \) are supplementary, then \( j \parallel k \).
Prove the Consecutive Interior Angles Theorem

**GIVEN:** $\angle 3$ and $\angle 5$ are supplementary

**PROVE:** $m \parallel n$

1. $\angle 3$ and $\angle 5$ are supplementary  
   1. Given

2. $\angle 5$ and $\angle 6$ are supplementary  
   2. Linear Pair Postulate

3. $\angle 3 \cong \angle 6$  
   3. Congruent Supplements Theorem

4. $m \parallel n$  
   4. Alternate Interior Angles Converse
EXAMPLE 2  Solve a real-world problem

Snake Patterns

How can you tell whether the sides of the pattern are parallel in the photo of a diamond-back snake?

SOLUTION

Because the alternate interior angles are congruent, you know that the sides of the pattern are parallel.
Can you prove that lines $a$ and $b$ are parallel? 

*Explain* why or why not.

3. Yes; Alternate Exterior Angles Converse.
Can you prove that lines $a$ and $b$ are parallel? Explain why or why not.

**Answer**
Yes; Corresponding Angles Converse.
Can you prove that lines \( a \) and \( b \) are parallel? \( Explain \) why or why not.

\[
5. \quad m \angle 1 + m \angle 2 = 180^\circ
\]

ANSWER

No; Supplementary angles do not have to be congruent.
EXAMPLE 4  Write a paragraph proof

In the figure, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$. Prove $p \parallel q$.

SOLUTION

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.
EXAMPLE 4  Write a paragraph proof

Plan for Proof

a. Look at $\angle 1$ and $\angle 2$.  

$\angle 1 \cong \angle 2$ because $r \parallel s$.

b. Look at $\angle 2$ and $\angle 3$.

If $\angle 2 \cong \angle 3$ then $p \parallel q$.  

EXAMPLE 4  Write a paragraph proof

Plan in Action

a. It is given that \( r \parallel s \), so by the Corresponding Angles Postulate, \( \angle 1 \equiv \angle 2 \).

b. It is also given that \( \angle 1 \equiv \angle 3 \). Then \( \angle 2 \equiv \angle 3 \) by the Transitive Property of Congruence for angles. Therefore, by the Alternate Interior Angles Converse, \( p \parallel q \).
THEOREM 3.7: Transitive Property of Parallel Lines
If two lines are parallel to the same line, then they are parallel to each other.

If $p \parallel q$ and $q \parallel r$, then $p \parallel r$. 
Prove the Transitive Property of Parallel Lines

GIVEN: \( p \parallel q \) and \( q \parallel r \)

PROVE: \( p \parallel r \)

1. \( p \parallel q \) and \( q \parallel r \)  \hspace{1cm} 1. Given
2. \( \angle 1 \cong \angle 2 \)  \hspace{1cm} 2. Alternate Interior Angles Theorem
3. \( \angle 2 \cong \angle 3 \)  \hspace{1cm} 3. Vertical Angles Congruence Theorem
4. \( \angle 3 \cong \angle 4 \)  \hspace{1cm} 4. Alternate Interior Angles Theorem
5. \( \angle 1 \cong \angle 4 \)  \hspace{1cm} 5. Transitive Property of Angle Congruence
6. \( p \parallel r \)  \hspace{1cm} 6. Alternate Interior Angles Converse
EXAMPLE 5  Use the Transitive Property of Parallel Lines

U.S. Flag

The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.
EXAMPLE 5  Use the Transitive Property of Parallel Lines

SOLUTION

The stripes from top to bottom can be named $s_1$, $s_2$, $s_3$, . . . , $s_{13}$. Each stripe is parallel to the one below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$. So, the top stripe is parallel to the bottom stripe.
6. If you use the diagram at the right to prove the Alternate Exterior Angles Converse, what GIVEN and PROVE statements would you use?

**ANSWER**

**GIVEN**: \( \angle 1 \cong \angle 8 \)

**PROVE**: \( j \parallel k \)
8. Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. *Explain* why the top step is parallel to the ground.

**ANSWER**

All of the steps are parallel. Since the bottom step is parallel to the ground, the Transitive Property of Parallel Lines applies, and the top step is parallel to the ground.
1. Find the value of $x$ that makes $p \parallel q$.

**ANSWER**  
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2. Can you prove $a \parallel b$? If so, what theorem would you use?

**ANSWER**  
Yes; Alternate Interior Angle Converse
Daily Homework Quiz

3. Which line are parallel.

**ANSWER** \( \overline{EF} \parallel \overline{DG} \)

4. In the figure, if \( \overline{HJ} \parallel \overline{KL} \) and \( \overline{KL} \parallel \overline{MN} \),

What can you conclude?

What theorem justifies your conclusion?

**ANSWER** \( \overline{HJ} \parallel \overline{MN} \) by Transitive Property of Parallel Lines
Closing

• Lines can be proved parallel by congruent corresponding angles, alternate interior angles, or alternate exterior angles. They can also be proved parallel if consecutive interior angles are supplementary.
• If two lines are parallel to the same line, they are parallel to each other.
Closing

You prove lines parallel by showing that corresponding angles, alternate interior angles, or alternate exterior angles are congruent, or by showing that consecutive interior angles are supplementary.
3.3 Homework

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